Angular momentum eigenstates of the isotropic 3-D harmonic oscillator: Phase-space distributions and coalescence probabilities

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The isotropic 3-dimensional harmonic oscillator potential can serve as an approximate description of many systems in atomic, solid state, nuclear, and particle physics. In particular, the question of 2 particles binding (or coalescing) into angular momentum eigenstates in such a potential has interesting applications. We have computed the probabilities for coalescence of two distinguishable, non-relativistic particles into such a bound state, where the initial particles are represented by generic wave packets of given average positions and momenta [1]. We have used a phase-space formulation and hence need the Wigner distribution functions of angular momentum eigenstates in isotropic 3-dimensional harmonic oscillators. These distribution functions have been previously discussed in the literature [2], but we utilized an alternative approach to obtain these functions. Along the way, we have derived a general formula that expands angular momentum eigenstates in terms of products of 1-dimensional harmonic oscillator eigenstates. As an example, Fig. 1 shows the coalescence probabilities P_{kl} for two Gaussian wave packets interacting through an isotropic 3-D harmonic oscillator potential as functions of relative coordinate r and momentum q, with probabilities that depend on the scalar product $\boldsymbol{r}\cdot\boldsymbol{q}$ are plotted for several values of the angle θ given by $\cos\theta = \boldsymbol{r}\cdot\boldsymbol{q}/rq$. The general wave-packet weighted Wigner functions derived in our

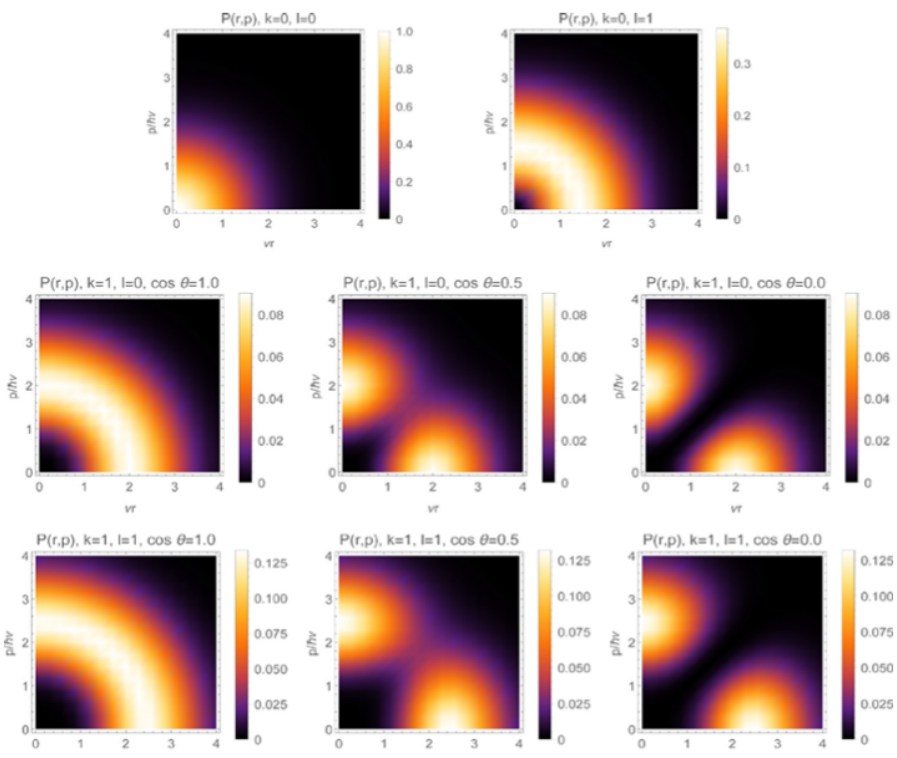

Fig. 1. Coalescence probabilities $\boldsymbol{P_{kl}}$, summed over m, for two Gaussian wave packets interacting through an isotropic 3-D harmonic oscillator potential as functions of relative coordinate r and momentum q, with probabilities that depend on the scalar product $\boldsymbol{r}\cdot\boldsymbol{q}$ are plotted for several values of the angle $\boldsymbol{\theta}$ given by $\boldsymbol{\cos\theta = r\cdot q/rq}$.

study allows us to extend the work in Ref. [3] to include the effect of radial excitations of hadrons on their production in relativistic heavy ion collisions.

[1] M. Kordell, R.J. Fries, and C.M. Ko, Ann. Phys. **443**, 168960 (2022).
[2] S. Shlomo and M. Prakash, Nucl. Phys. **A357**, 157 (1981).